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WEB EXCLUSIVES

Developing Time Horizons for Use in Portfolio Analysis

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This article provides a time-referenced perspective to portfolio analysis. Time-horizon modeling should be an integral component of any portfolio's investment policies and practices. Risk and return take on probabilistic meanings, while asset streams must be balanced with and fund the liability streams. Instead of investment trade-offs being limited to risk and return only, the investing experience is seen as a more sophisticated and complex mix of risk, return, time and liability concepts.

"It might seem puzzling why the holding period has almost never been considered in portfolio theory . . . The holding period becomes a critical issue in portfolio theory when data reveal the mean reversion of stock returns."

—Jeremy Siegel, 1998

This wonderful quote by a distinguished economics professor sums up the current state of investment analysis. For over 50 years, portfolio theory has typically assumed a unitary time horizon in calculating optimized levels of risk and return. A lack of time "consciousness" may be tolerable for short-term traders and near-term investors who do not hold investments for any length of time. In institutional and pension settings however, using a timeless horizon is simply untenable.

For portfolios having both asset and liability streams lasting a generation or more, the relevant holding period becomes crucial. When evaluating risk and return concepts, the following inquiries should be made:

- In what time frame?
- What are the probabilities of return?
- What are the liabilities?

In What Time Frame?

Portfolio risk and return is typically calculated through means-variance optimization (MVO) techniques. A unitary time horizon is normally assumed, with short-term data being used to generate portfolio rates of return and standard deviation. An annualized rate of return using one-year holding periods (or less) is quite common. This produces a very high level of pricing volatility for various asset classes, especially when historical data over the course of the modern era of investing is referenced. For instance, equity standard deviation estimates of 16% to 20% are commonly generated when using historical data from 1926 to the present.

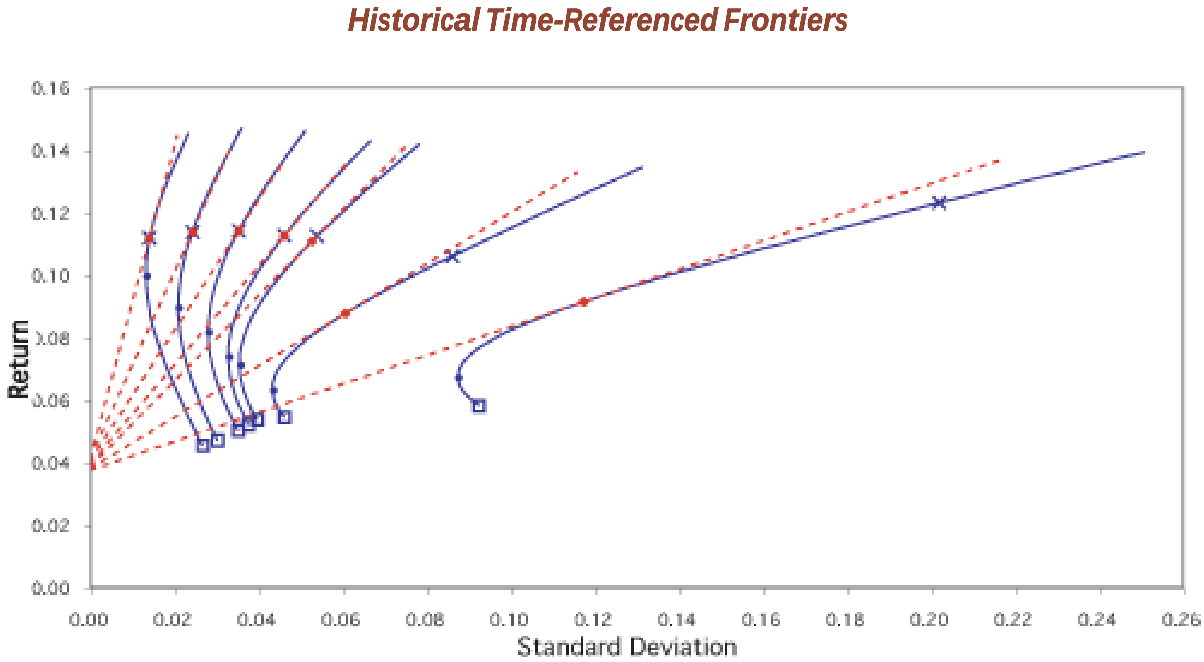
The first question that should be entertained is the intended time frame of the investment or portfolio being studied. Siegel (1998), Ibbotson (2005), and many others have shown that pricing volatility is remarkably reduced in long time frames. The basic procedure is to calculate the annualized rate of return for the various asset classes over successively longer holding periods. A series of efficient frontiers is generated, one for each time frame. Optimized levels of risk and return, as well as optimal asset allocations, are then calculated for each holding period.

Figure 1 displays time-referenced frontiers and points of optimality for a portfolio composed of equity and bond asset classes. The inspiration for the figure comes from Siegel (1998), using different data tracts going back to the 1800s. Optimal levels of risk and return for historical periods are identified in Figure 1, with the one-year frontier sketched on the right side of the graph, followed by, from right to left, frontiers for 5, 10, 15, 20, 25 and 30 holding periods. The small boxes at the bottom of each frontier represent a 100% bond position, while the "x" on the frontiers is a 100% equity position.

Figure 1 visually demonstrates the relevance of time horizons. Optimal levels of return increase slightly over time, largely due to the increasing equity allocations in successively longer time frames. Standard deviation is almost eliminated over time, the result of pricing returns reverting to their mean average. As John Bogle (1999) so eloquently stated, "descending the slope of risk is far easier than ascending the slope of reward."

A simple buy and hold position of the market portfolio will rather automatically

Figure 1



extinguish pricing risk over the course of time. The inquiry into the applicability of time horizons is only beginning, however, and is dependent upon several other items, which are discussed below.

What Are the Probabilities of Return?

Standard portfolio textbooks treat the optimal levels of risk and return as the derived output of the means-variance process. While that is mathematically correct, what often becomes lost in the discussion is that portfolio return is merely the mean average, or 50th percentile, of an entire probability distribution of return data. Rather than using an expected rate of return, it would be far better to establish a range of returns over defined time frames and designated probabilities. This would result in providing trustees and administrators of plans not only mean average information, but also projections at the extremes.

It is quite helpful to view return as a series of probability distributions across time, with the rate of return calculated to a certain degree of confidence. This is consistent with research that examines

Figure 2

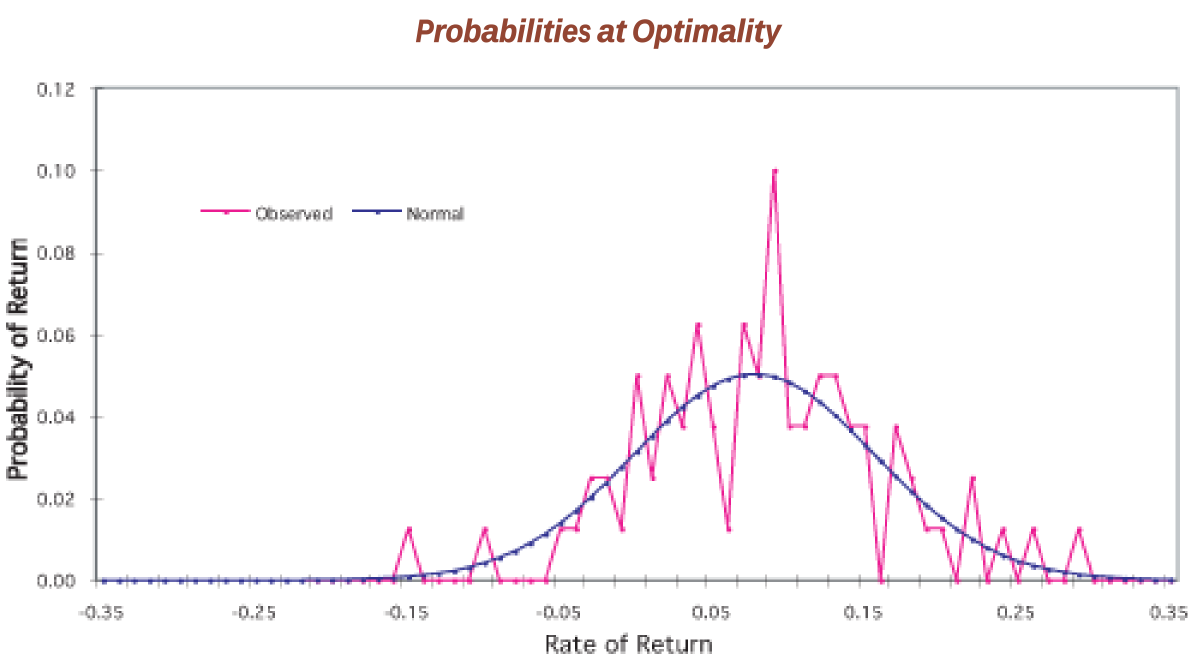
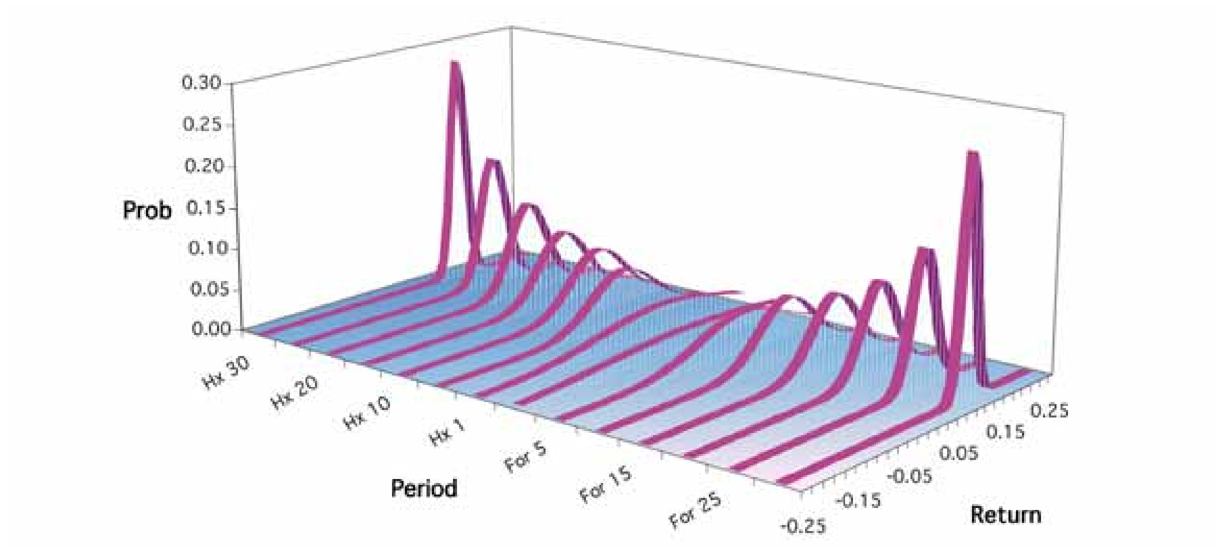


Figure 3

Discrete Return Probabilities at Optimality



the probabilities of return data under normal market conditions, to a certain confidence level (Jorion, 2001). By viewing risk and return in probabilistic terms, the likelihood of a certain level of return, either on the average or at the extremes, can be produced for any given holding period.

Probability distributions of return can be developed for each holding period.

Using standard means-variance optimization techniques, the following probability distribution for historical return data was generated. See Figure 2.

Optimal combinations of risk and return and probability distributions at optimality can be similarly arrived at for all historical holding periods. Forward periods can also be provided for, by using a mix of historical data and analyst projections to estimate the probabilities of return. The probability distributions of optimal rates of return and risk for each holding period can then be visually presented within a three-dimensional context. Rates of return, the intended holding period and the probability of return can be assigned to designated x-y-z coordinates. Optimality using both past historical data and forward estimations can be incorporated into one graph, as in Figure 3.

A series of discrete return distributions displayed across time is produced in Figure 3. Note that holding periods are used, instead of exact time sequences, as the probability distributions portray the risk-return trade-off across successively longer holding periods. Historical periods are on the left side, while forward periods are on the right side. Both sides can be viewed as being forward looking in nature, however. The left side depicts a future scenario that is exactly the same as

Figure 4

Continuous Return Probabilities at Optimality

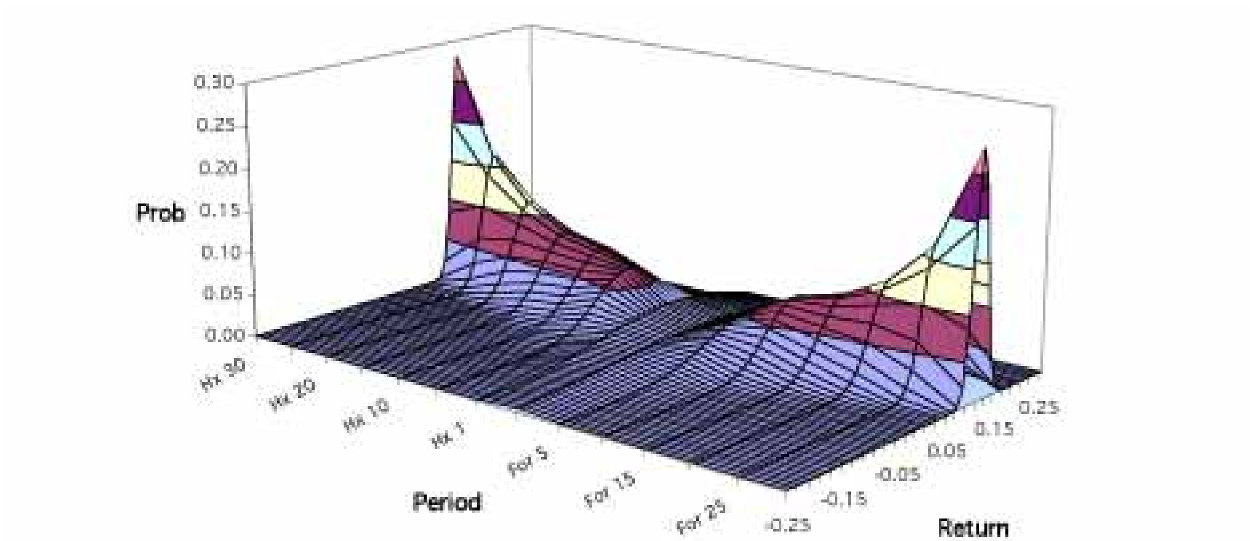


Figure 5

Return Percentiles at Optimality

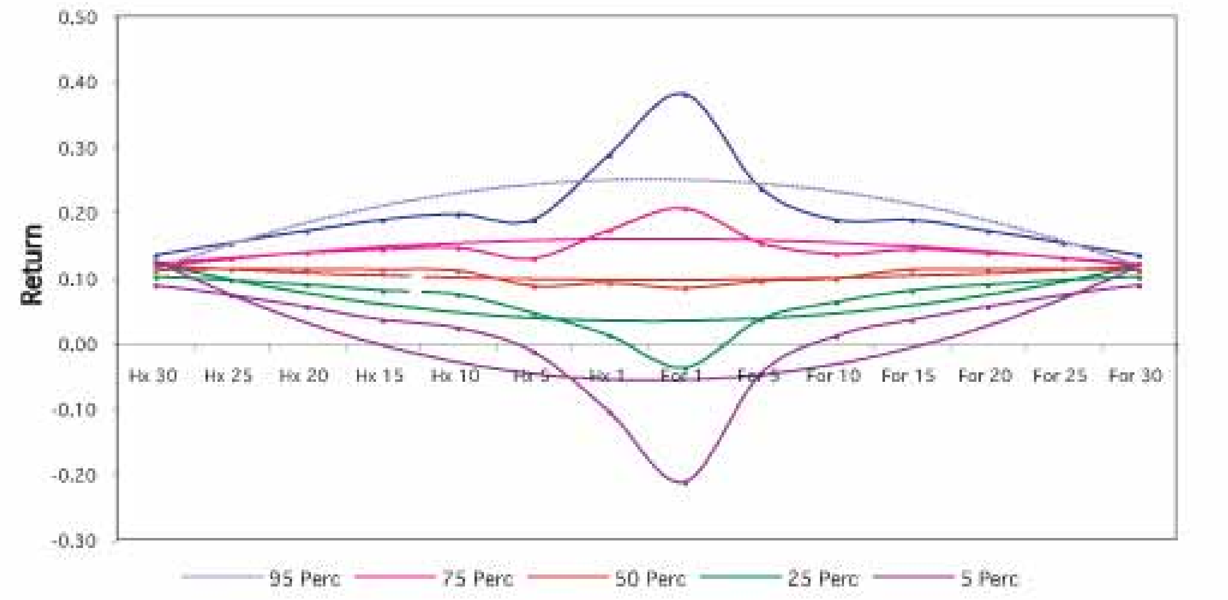
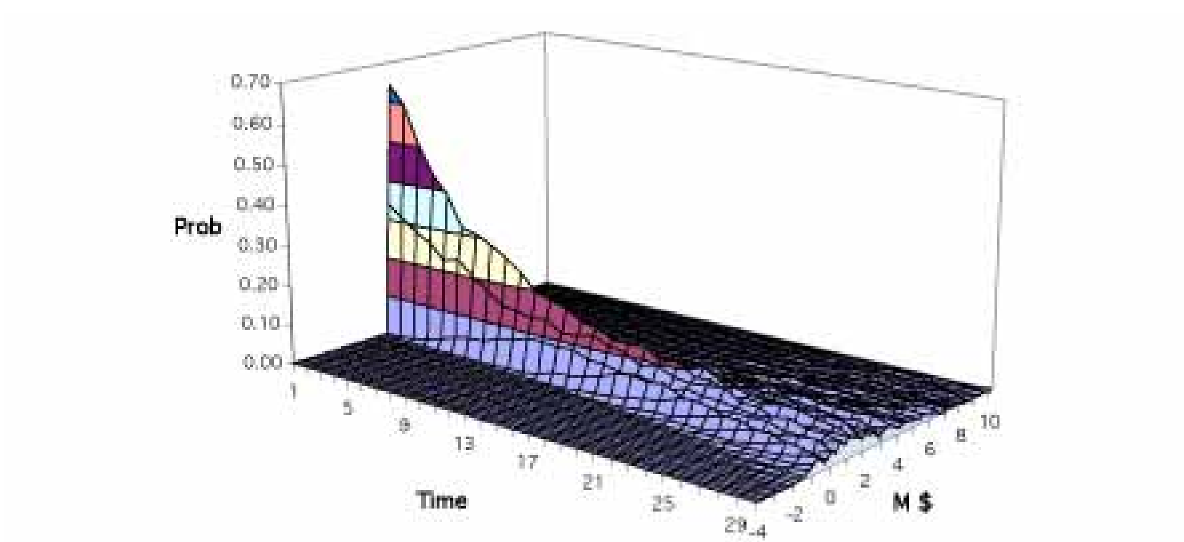


Figure 6

Portfolio Net Wealth Probabilities



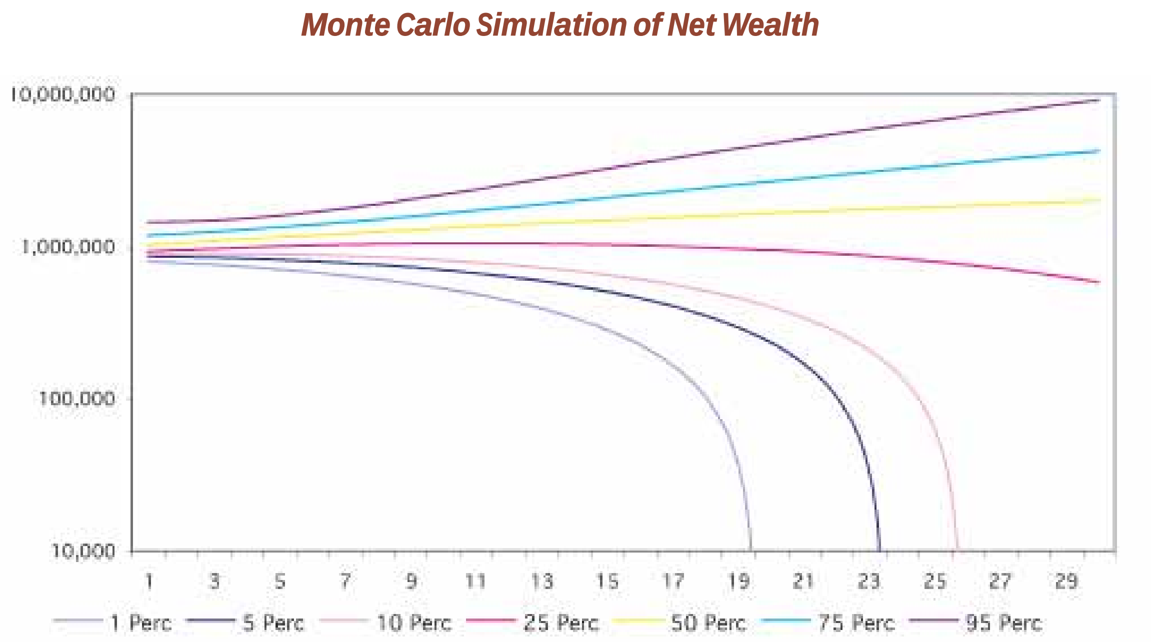
the past, while the right side contains forward estimations that may deviate off of past historical events. The two sides can thus be seen as mirror images of each other, with some changes occurring between historical and forward projections.

The discrete probability distributions can be translated into a continuous probability function or envelope, as shown in Figure 4. All holding periods are now contained in the figure, through a continuous relationship between the probabilities of return, the holding period and expected rates of return. (See Figure 4.)

The probability distribution envelope can also be stated as a range of returns, to a certain degree of confidence. This is a very important exercise, since we can now estimate the confidence that we have in the return estimates. Figure 5 displays the optimal portfolio return probabilities, only in terms of levels of confidence (and done in a two-dimensional format). The solid lines connect the single point estimates for each holding period, while the dotted curves are generated from 2nd order polynomial trend-lines, representing smoothed confidence levels. At the 5th and even the 25th percentiles, returns can be expected to be negative in the shorter time horizons, both historically and on a forward-looking basis.

The slight dip in the five-year historical single point estimate is the result of the optimization equations generating greatly reduced risk levels on the five-year frontier, while the comparatively low rate

Figure 7



of return is the result of the frontier not yet rotating upward. The dip in the forward five-year horizon is from projected underperformance in near-term returns, as well as the forward five-year frontier also not yet rotating upward, either. If long-term return projections were used for all forward-holding periods, the returns at the various confidence levels would be more uniform. Additionally, if asset allocations were incrementally adjusted instead of quickly changing by way of the optimization equations themselves, smoother estimations of portfolio return probabilities across time (both historical and forward) would be produced.

The importance of the above probability functions lies not in their mathematical and statistical properties however, but in one very important conceptual realization: The investing experience exists along the probability boundaries. In traditional MPT analysis, an investor can attain, but not exceed, the risk and return choices that lie on the efficient frontier. Anything to the right of the frontier is considered an inferior choice. Anything to the left of the frontier is unattainable. Investor marginal utility is maximized along the frontier, and more specifically, at the tangency between the Capital Market Line and the efficient frontier. The same analysis can be applied to probability distributions across time horizons. At optimality, an investor can attain a certain probability of return in a designated time frame. A higher probability of return beyond the contours of the envelopes is not possible. A lower return probability is attainable, but would be inferior to the probability envelope. Marginal utility will be maximized along the return probability boundaries.

What Are the Liabilities?

Portfolio theory typically goes into great detail discussing the asset side of the investment process. But investment analysis is nearly devoid of thoughts regarding the liability side. For instance, the first paper on shortfall risk (A.D. Roy, 1952) only warrants a footnote in many portfolio texts, if there is any reference at all. The cash flow withdrawal stream of a mature portfolio is of vital concern however, to any investor contemplating retirement. Arguably, analysis of investment assets is simply incomplete without a corresponding analysis of the investment-related liabilities.

Means-variance models can be extended to include liability streams. The following figures are generated from a Monte Carlo simulation of a $1 million portfolio having annual cash withdrawals of $50,000 per year plus an inflation adjustment. The equations, procedures and assumptions of the simulation closely follow and emulate an example provided by Ibbotson (2005). Figure 6 simulates the range of wealth of 500 portfolios at various percentiles, and is done within a three-dimensional context of time (x), return (y) and probabilities of return (z). Fixed allocations across all time periods of 40% equities, 60% bonds were assumed for the simulation. Once initial estimations of shortfall are arrived at, the results can be fed back into the process, thereby producing an adjustment of allocations to more fully achieve investment objectives.

When liabilities are added to the analysis, the shortfall from stated investment objectives and goals becomes the relevant risk to investors. While this can be visually seen in the above graph as anything below zero, the probability of shortfall becomes more apparent when the same data is placed on a logarithmic scale, as is done in Figure 7. The figure is in close accord with a graph contained in Ibbotson (2005), where 5,000 trials were conducted on varying asset allocations.

Notice the remarkable spread of portfolio values in the longer time frames. Of particular concern is the number of portfolios failing to adequately fund the retirement investment objective. The probability of shortfall is displayed as portfolios in decline at given probability confidence levels. Portfolio failure occurs at the one percentile level of confidence by year 20; the 5th percentile by year 23, and the 10th percentile by year 26. Even the 25th percentile is in decline by year

30, and will ultimately fail, given enough time. To alleviate this problem posed with the assumed facts of the simulation, allocations should be adjusted to reduce the percentile of portfolios failing. If allocation changes fail to reduce shortfall probabilities to levels deemed to be more appropriate by the trustees, then funding policy changes and benefit adjustments (where not in violation of anticutback rules) could then be contemplated.

Figure 7 demonstrates that as pricing variance drops out of long-term investment considerations, the risk of shortfall in meeting investment objectives increases markedly. This demonstrates the critical nature of liability streams in formulating appropriate investment policies. Focusing on the optimization of pricing risk and return without considering the ever-increasing spread of terminal net wealth and the possibility of shortfall at the extremes is tantamount to a major analytical blunder.

For long-term investors, the meaning and measure of risk and return shifts over time. No longer are investors primarily concerned with portfolio returns and the volatility of those returns. Pricing returns and the deviation off of expected returns are only relevant in the context of long-term probabilities of shortfall. Will there be enough to live on during retirement? That is the real risk of the long term.

Conclusion

By extending portfolio theory to include time horizons, more effective and sophisticated investment policies can be developed. Risk and return become probabilistic in nature, changing in measure and meaning over the course of time. Investor-level risk preferences dramatically affect allocation decisions, resulting in a more complex set of investment choices. Neither market-level optimality nor individual-level factors can be fully appreciated and evaluated without integrating time, the probabilities of return and liabilities into the investment process. Investment policies and practices should therefore consider and include risk, return, time and liability concepts. B&C

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